Helwan University
Faculty of Computers and Information

(Computer Exercise)

Department of Computer Science
Image Processing

1. Prove that

$$\langle \cos(nt), \cos(mt) \rangle = \pi \delta_{nm},$$
$$\langle \sin(nt), \sin(mt) \rangle = \pi \delta_{nm},$$
$$\langle \sin(nt), \cos(mt) \rangle = 0$$

2. Prove that for any function f, defined over any period T,

$$\widehat{\beta}_0 = \frac{1}{T} \int_T f,$$
$$\widehat{\beta}_n^{(e)} = \frac{2}{T} \int_T f(t) \cos\left(\frac{2\pi}{T} nt\right) dt,$$
$$\widehat{\beta}_n^{(o)} = \frac{2}{T} \int_T f(t) \sin\left(\frac{2\pi}{T} nt\right) dt.$$

3. Consider the function $f(t),\ t \in [-\pi, \pi], f(t) = \begin{cases} 1 & -\pi/2 < t < \pi/2 \\ 0 & \text{otherwise} \end{cases}$.

- Find the Fourier expansion up to harmonic N.
- Plot $\widehat{f}(t)$ for $N = 1, \ldots, 20$; i.e., 20 figures. Observer how $\widehat{f}$ gets closer to f for higher N
- Plot the error $\|e\| = \sqrt{\left\langle f - \widehat{f}, f - \widehat{f} \right\rangle}$ for each N.